Neuromorphic circuits: nonlinearity & neuristors

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Director, Foundational Technologies
Background

HP Labs Foundational Tech Group
photonics & nanoelectronics
Systems Integration Group
Systems Software Group
Cognizant systems requires convergence of disciplines

Cognizant Systems

Software Platform
Cog ex Machina

Quantitative Psychology

Adaptive Resonance Theory

Computer Science

Hardware Accelerators
nano-enabled cores

Neuro-physiology

New Devices
memristors & neuristors

Hybrid Integrated Circuits
multi-functional multi-core chips
Hybrid Technology System in a Package

Compute with Electrons
‘Standard’ Processor Cores
Accelerators using new devices:
crossbar algebra engines
neuristors (learning and decisions)
more functionality per transistor

Remember with Ions
Data Storage (3D memristor crossbars)

Silicon Photonic Fabric
Communicate with Photons

No ‘exotic’ materials, processes or environments!
2 memristor crossbar arrays on top of CMOS

H. D. Lee et al., 2012 Symposium on VLSI Technology
Take-away messages

• Biological circuits and signals are highly nonlinear
  • nonlinear network theory not familiar or utilized
  • high information content (important or not?)
  • ‘edge of chaos’ – necessary or coincidental?
• A neuristor is an electronic analog of a HH neuron
  • electron channels (memristors) and capacitors
  • action potential, rich spiking behavior
  • CMOS compatible materials and processes
• The ‘transistor equivalent’ for neuromorphic circuits?
How can nonlinear network analysis help with understanding the connectome?

Much of the work with which I am familiar is implicitly based on linear networks.

What is conceptually missing?
Delta-Wye Transform Is Not Valid for Nonlinear Devices!

Nonlinear devices are not commutative!
Introduction to nonlinear network theory (1969)
Leon Chua

Ten years of research - 987 pages!

Very few engineers utilize the methods or concepts:

book out of print for ~30 years
too much material to swallow – no ‘easy’ introduction
SPICE handles nonlinear systems numerically –
but does not yield intuition
circuit designers avoid nonlinearity for fear of chaos
A Memristor (closely) Obeys Chua’s Equations:

\[
v = R(w, i)i
\]

Quasi-static conduction eq. – Ohm’s Law

\[
\frac{dw}{dt} = f(w, i)
\]

Dynamical eq. – evolution of state


\(w\) is the state variable (or variables)

\(w\) describes the physical properties of the circuit element

Examples (any property that changes material resistance):

- oxygen vacancy (dopant) concentration in oxides
- structural phase (amorphous vs. crystalline)
- magnetization or spin state
- correlated electron state (Mott insulator vs. conductor)
What is a Locally Active Memristor?

Displays Negative Differential Resistance

Memory can be transient (volatile)

e.g. Mott Insulator to Metal Transition
Mott Insulator to Metal Transition in Oxides Produces Negative Differential Resistance Device

Resistance vs. Temperature for VO$_2$
Kumar et al., unpublished

Geppert, Proc. IRE, 1963
Chopra, Proc. IRE, 1963
Locally Active Memristor Model for Thermally-Induced NDR

Pickett and Williams, Nanotechnology 23 (2012) art.# 215202

\[ i = G(u)\nu, \quad u = \frac{r_{\text{met}}}{r_{\text{chan}}} \text{ (state variable)} \]

\[ \frac{du}{dt} = \left(\frac{d\Delta H}{du}\right)^{-1}(G(u)\nu^2 - \Gamma(u)\Delta T) \]

transition enthalpy

Joule heating

heat loss

Rate (+ and −) limited by heat capacity
Potassium Ion Channel
(transport of $K^+$ out of the axon membrane)
structural and mechanistic properties

Roderick MacKinnon, MD
Chemistry Nobel Prize, 2003
http://www.osti.gov/accomplishments/mackinnon.html

David Clapham, MD PhD
http://www.childrenshospital.org/dream/summer2003/body.html
Locally Active Memristor Model for NbO$_2$ NDR

Oscillator with DC bias!

Current (x100 $\mu$A) vs. Voltage (V)

$r_{ch} = 30$ nm

Current (x100 $\mu$A) vs. Time ($\mu$s)

$b)$

(c)
Performance of NbO$_2$ locally active memristor

For our devices:

channel radius = 30 nm
$\Delta T = T_{\text{trans}} - T_{\text{amb}} = 800$ K
$\Delta t_{\text{ON}} = 0.7$ ns & $\Delta H = 60$ fJ

Smaller size, alternate material:

channel radius = 10 nm
$\Delta T = T_{\text{trans}} - T_{\text{amb}} = 200$ K
$\Delta t_{\text{ON}} = \sim 10$ ps & $\Delta H = \sim 6$ fJ
What is a Neuristor?


Previous implementations required inductors.
Leon Chua’s Revision of the Hodgkin-Huxley Model

Removes anomalies and paradoxes

L. Chua et al., “Hodgkin-Huxley Axon is made of Memristors,”
A. J. Cote proposed the first prototype neuristor in 1961, and Jin-ichi Nagumo demonstrated an alternate in 1962. Size of a shoebox. Very little work since.

Four state variables: $q_1, u_1, q_2, u_2$
Neuristor Pulse Amplification and Threshold Behavior

(a) Super-threshold Input $\Delta V = 0.3 \, V$

(b) Super-threshold Output $\Delta V = 0.53 \, V$

(c) Sub-threshold Input $\Delta V = 0.2 \, V$

(d) Sub-threshold Output $\Delta V = 28 \, mV$
Neuristor Equivalent Circuit Diagram

\[ \Delta q_1 \quad \Delta q_2 \]

\[ C_1 \quad C_2 \]

\[ M_1 \quad M_2 \]

\[ V_1 \quad V_2 \]

\[ R_L \quad \text{input} \quad \text{output} \]

Action potential

\[ u_1 \quad \Delta q_1 \quad \Delta q_2 \quad V_{out} \quad t (\mu s) \]

\[ -600 \quad -300 \quad 0 \]

\[ -150 \quad -75 \quad 0 \quad 75 \]
Neuristor Spiking emulates signals seen in brains

“Regular Spiking”
\[ C_1 = 5.1 \text{ nF}, \quad C_2 = 0.75 \text{ nF} \]

“Chattering”
\[ C_1 = 5.1 \text{ nF}, \quad C_2 = 0.5 \text{ nF} \]

“Fast Spiking”
\[ C_1 = 1.6 \text{ nF}, \quad C_2 = 0.5 \text{ nF} \]
Compare bifurcation diagrams: neuristor and HH neuron

Chua has noted that biology is poised on the ‘edge of chaos’—
High information or low power?

“fast spiking” neuristor

HH axon
Chua et al., 2012
Neuristor Summary – an electronic and inorganic HH unit

We have built a Neuristor using two Locally-Active Memristors and two Capacitors as the Dynamical Elements.

The Neuristor has four state variables: the radii of the conduction channels of the memristors and the capacitor charges.

The NbO$_2$ Memristors exhibit an insulator-to-metal transition that mimics the opening and closing of axon ion channels.

We have observed signal gain, thresholding, attenuation and spiking that closely mimics the behavior of neuron action potentials.

We are designing and simulating transistorless digital logic and analog circuits based on neuristor units using standard CMOS tools.
Neuristor Computation

Boolean Logic Gates

Rule 137 Cellular Automaton
Neuristor Spike-Based Circuit Elements
Neuristor Spike-Based AND and NOT Gates

(a) AND

(b) NOT

(c) Voltage (V)

(d) Output Height (V)

(e) NOT

(f) ASRT

(g) Voltage (V)

(h) Output Height (V)
Description of Rule 137 Cellular Automaton

Leon O. Chua

World Scientific
Logical Design of Rule 137 Cellular Automaton

(a) rule 137 cell

To i-1

To i

To i+1

From i-1

V_{in,-1}

From i

V_{in,0}

From i+1

V_{in,+1}

A (2τ)  B (1τ)  C (2τ)  D (1τ)  E (2τ)  F (3τ)  G (2τ)

(b) r137

r137 1   r137 2   r137 3   r137 4   r137 5   r137 6

IF(0,0,0)  OR

READ

SET INITIAL

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Operation of Rule 137 neuristor cellular automaton

Truth Tables for CA Rules 110 and 137

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$2^8 = 256$ different rules
Future Research Directions

Understand insulator to conductor transitions!
Predict materials with desired properties
Find transition temperature of ~200 °C

Neuromimetic integrated circuits –
Using nanowire capacitance (attoFarads),
potential for very high speed and low power circuits -
stand-alone or CMOS hybrid

Not attempting to replace Si transitors,
but complement them