Neuromorphic circuits: nonlinearity & neuristors

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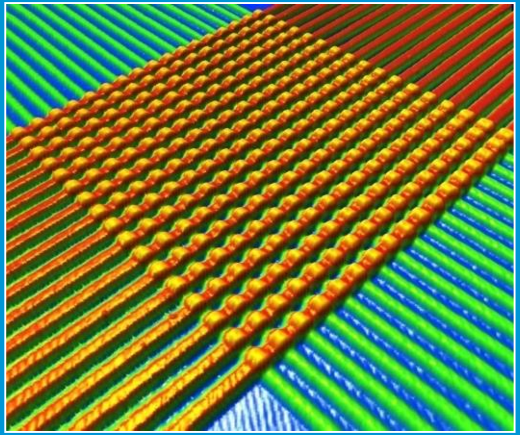


Background

HP Labs Foundational Tech Group

photonics & nanoelectronics

Systems Integration Group

Systems Software Group

Cognizant systems requires convergence of disciplines

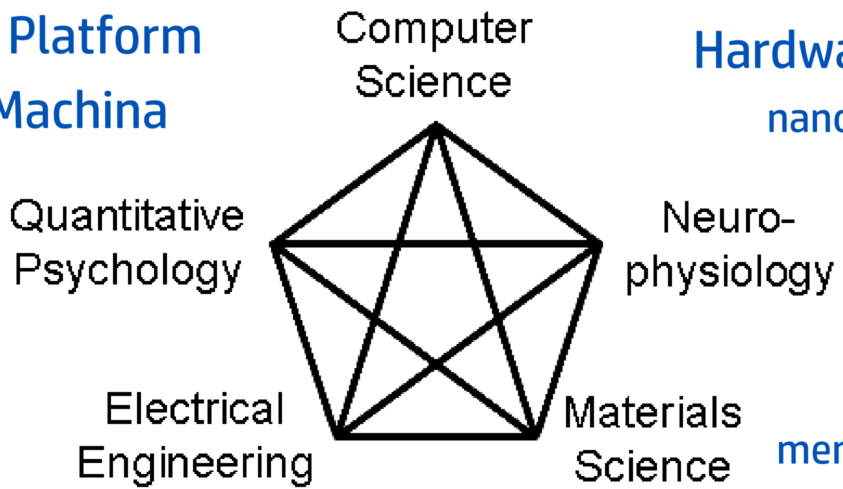
Cognizant Systems

Software Platform

Cog ex Machina

Hardware Accelerators

nano-enabled cores



Adaptive Resonance Theory

New Devices

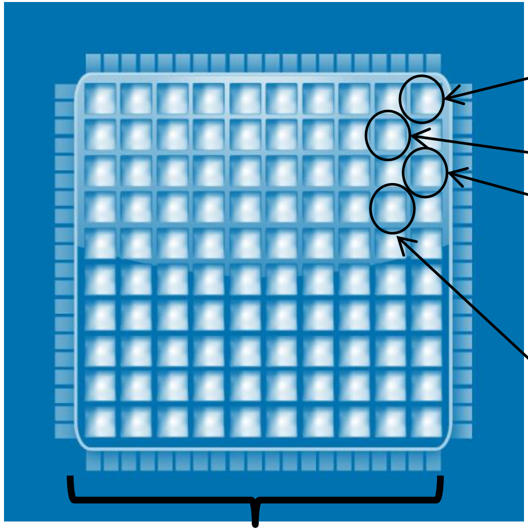
memristors & neuristors

Hybrid Integrated Circuits

multi-functional multi-core chips

Hybrid Technology System in a Package

multi-level package



Silicon Photonic Fabric

Communicate with Photons

Compute with Electrons

'Standard' Processor Cores

Accelerators using new devices:

- crossbar algebra engines
- neuristors (learning and decisions)

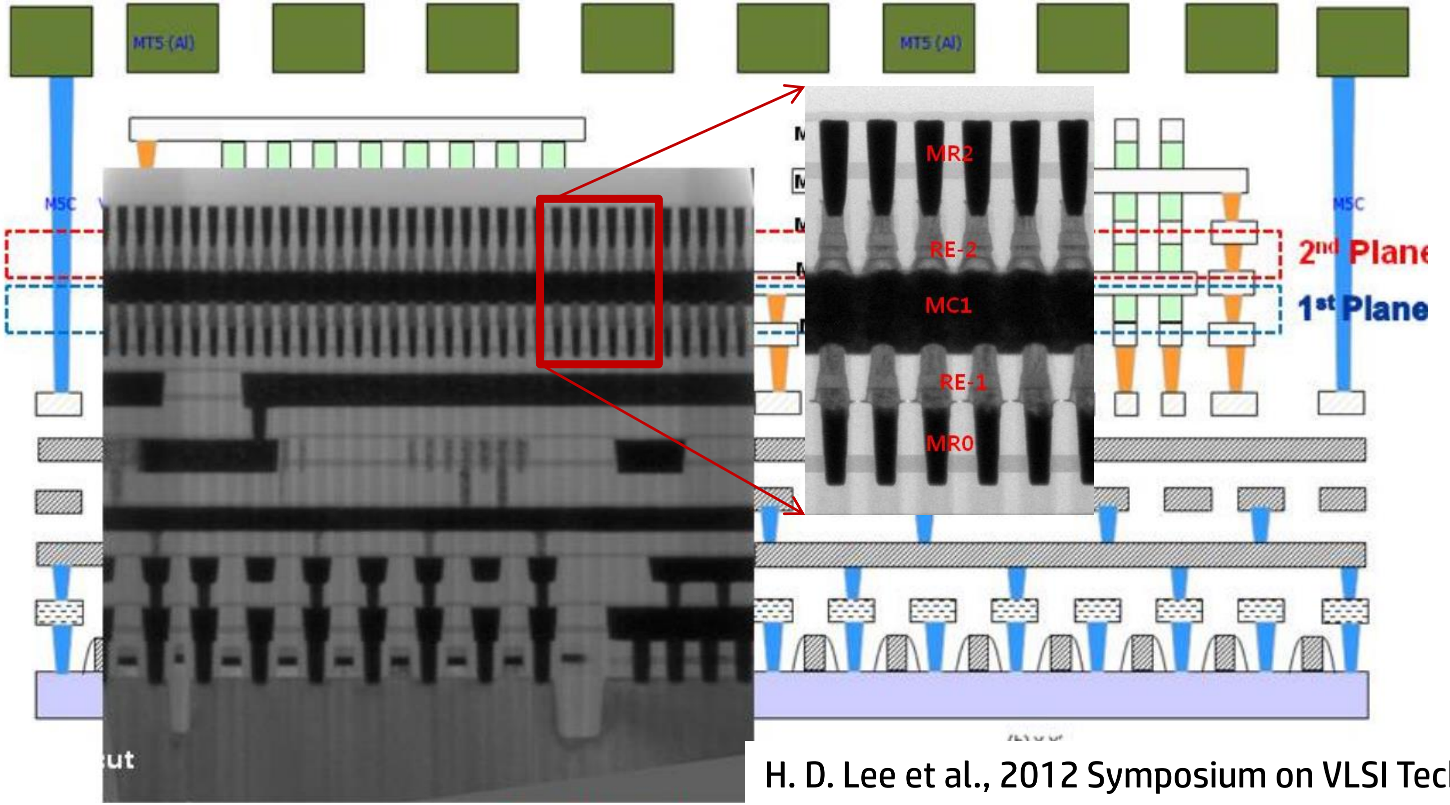
more functionality per transistor

Remember with Ions

Data Storage (3D memristor crossbars)

No 'exotic' materials, processes or environments!

2 memristor crossbar arrays on top of CMOS



H. D. Lee et al., 2012 Symposium on VLSI Technology

Take-away messages

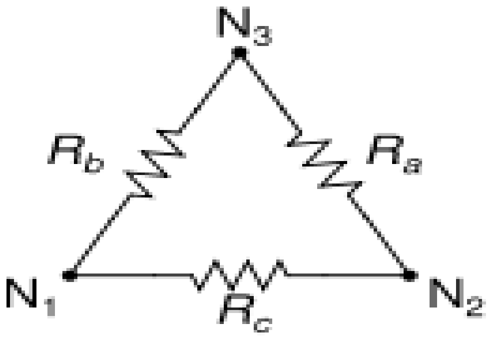
- Biological circuits and signals are highly nonlinear
  - nonlinear network theory not familiar or utilized
  - high information content (important or not?)
  - ‘edge of chaos’ – necessary or coincidental?
- A neuristor is an electronic analog of a HH neuron
  - electron channels (memristors) and capacitors
  - action potential, rich spiking behavior
  - CMOS compatible materials and processes
- The ‘transistor equivalent’ for neuromorphic circuits?

How can nonlinear network analysis help with understanding the connectome?

Much of the work with which I am familiar is implicitly based on linear networks

What is conceptually missing?

Delta-Wye Transform Is Not Valid for Nonlinear Devices!



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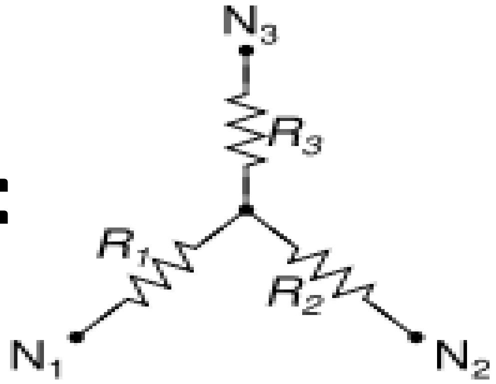


$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1},$$
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2},$$
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c},$$
$$R_2 = \frac{R_aR_c}{R_a + R_b + R_c},$$
$$R_3 = \frac{R_aR_b}{R_a + R_b + R_c}.$$

Nonlinear devices are not commutative!

Introduction to nonlinear network theory (1969)
Leon Chua

Ten years of research - 987 pages!

Very few engineers utilize the methods or concepts:

- book out of print for ~30 years
- too much material to swallow – no ‘easy’ introduction
- SPICE handles nonlinear systems numerically –
  but does not yield intuition
- circuit designers avoid nonlinearity for fear of chaos

A Memristor (closely) Obeys Chua's Equations:

$$v = R(w,i)i$$ Quasi-static conduction eq. – Ohm's Law

$$\frac{dw}{dt} = f(w,i)$$ Dynamical eq. – evolution of state

L. O. Chua and S. M. Kang, "Memristive devices and systems," *Proc. IEEE,* 64 (2), 209-23 (1976).

w is the <u>state</u> <u>variable</u> (or variables)

w describes the physical properties of the circuit element

Examples (any property that changes material resistance):

oxygen vacancy (dopant) concentration in oxides

structural phase (amorphous vs. crystalline)

magnetization or spin state

correlated electron state (Mott insulator vs. conductor)

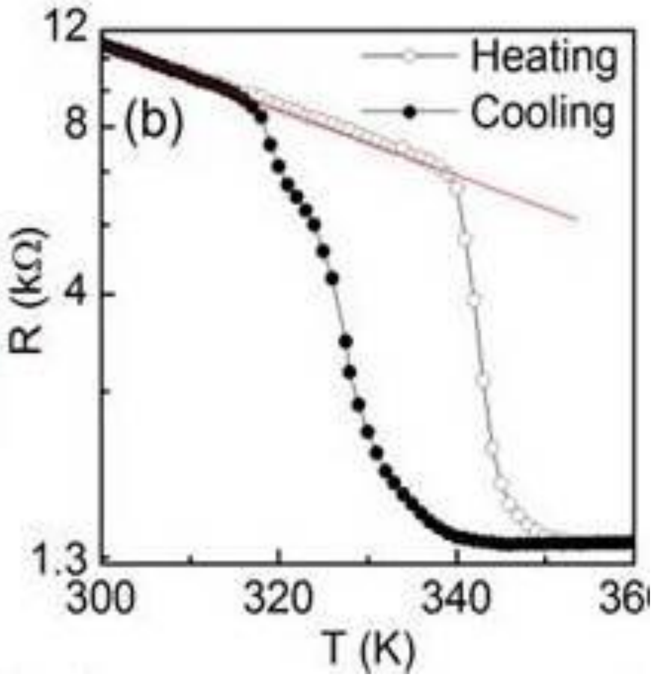
What is a Locally Active Memristor?

Displays Negative Differential Resistance

Memory can be transient (volatile)

e.g. Mott Insulator to Metal Transition

Mott Insulator to Metal Transition in Oxides Produces Negative Differential Resistance Device



Resistance vs. Temperature for VO_2
Kumar et al., unpublished

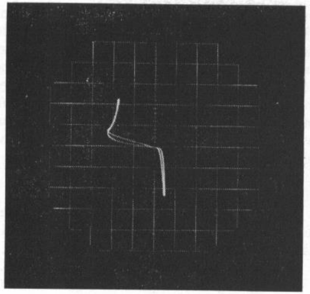


Fig. 1—Current-voltage trace of niobium oxide using mercury counter-electrode. Vertical scale 0.6 ma/division, horizontal scale 5 volts/division. The negative resistance region corresponds to the mercury being negative relative to the niobium.

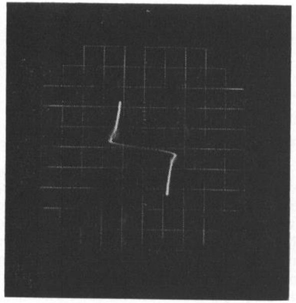


Fig. 2—Symmetrical current-voltage trace of niobium oxide using probe counter-electrode. Parasitic oscillations are visible in the negative-resistance regions. Vertical scale 0.1 ma/division, horizontal scale 5 volts/division.

Geppert, *Proc. IRE*, 1963

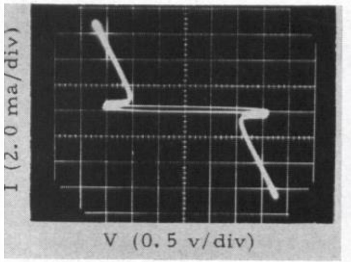


Fig. 1—I-V characteristics at 78°K with a negative resistance region.

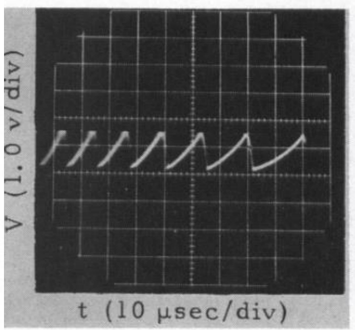


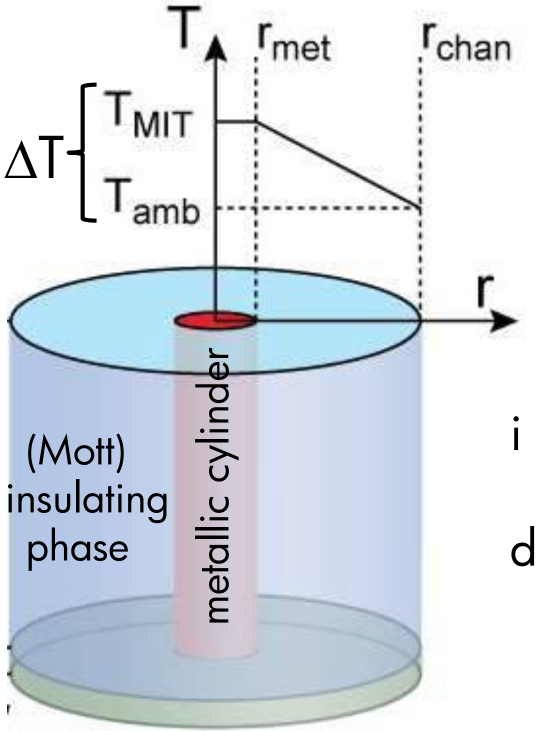
Fig. 3—Trace of sustained oscillations in the negative resistance region.

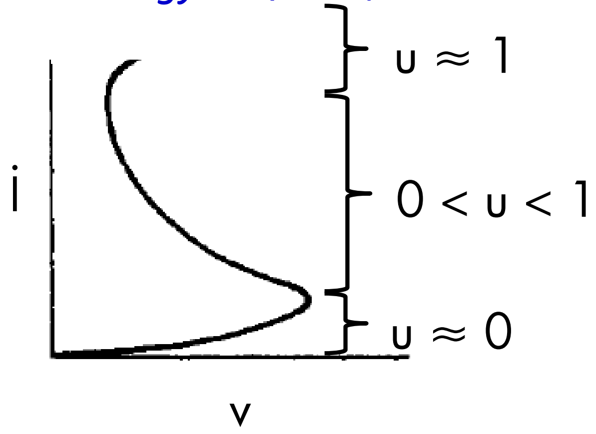
Chopra, *Proc. IRE*, 1963

Locally Active Memristor Model for Thermally-Induced NDR

Pickett and Williams, *Nanotechnology* **23** (2012) art.# 215202

‘electron valve or channel’





$i = G(u)v$, $u = r_{met}/r_{chan}$ (state variable)

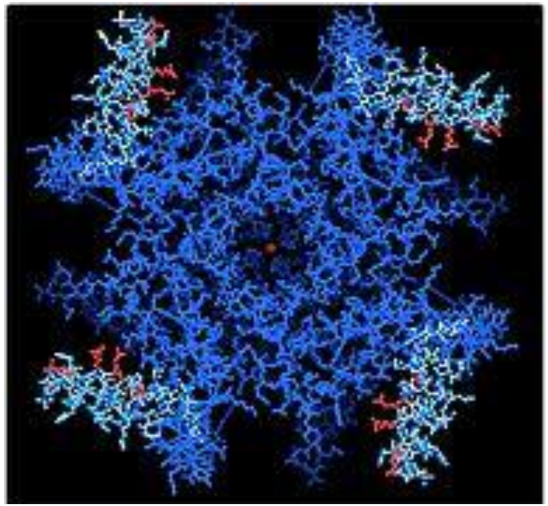
$$du/dt = (d\Delta H/du)^{-1}(G(u)v^2 - \Gamma(u)\,\Delta T)$$

- $(d\Delta H/du)^{-1}$: transition enthalpy
- $G(u)v^2$: Joule heating
- $\Gamma(u)\,\Delta T$: heat loss

Rate (+ and –) limited by heat capacity

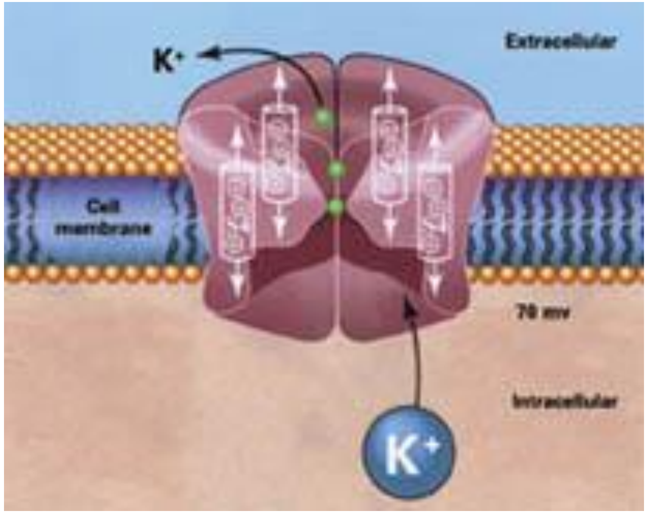
Potassium Ion Channel
(transport of K^+ out of the axon membrane)
structural and mechanistic properties

‘ion valve’



Roderick MacKinnon, MD
Chemistry Nobel Prize, 2003

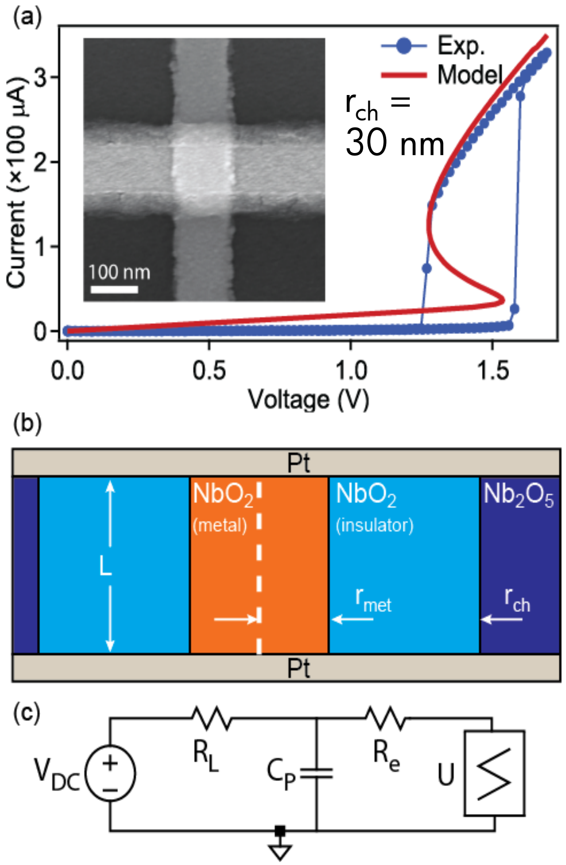
http://www.osti.gov/accomplishments/mackinnon.html



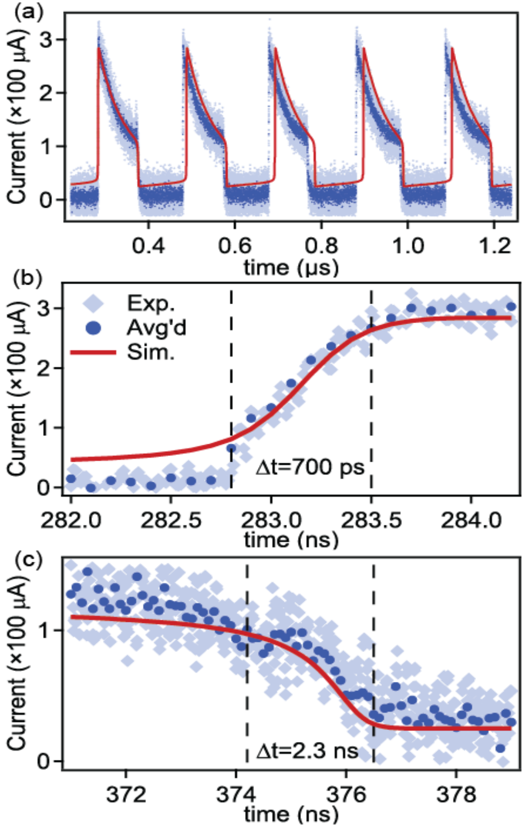
David Clapham, MD PhD

http://www.childrenshospital.org/dream/summer2003/body.html

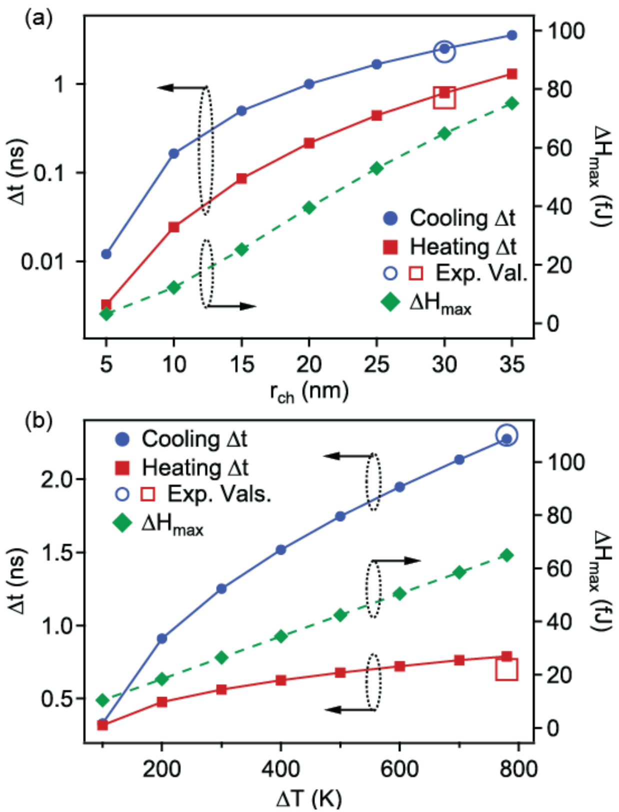
Locally Active Memristor Model for NbO_2 NDR



Oscillator with DC bias!



Performance of NbO_2 locally active memristor



For our devices:

channel radius = 30 nm

$\Delta T = T_{trans} - T_{amb} = 800$ K

$\Delta t_{ON} = 0.7$ ns & $\Delta H = 60$ fJ

Smaller size, alternate material:

channel radius = 10 nm

$\Delta T = T_{trans} - T_{amb} = 200$ K

$\Delta t_{ON} = \sim 10$ ps & $\Delta H = \sim 6$ fJ

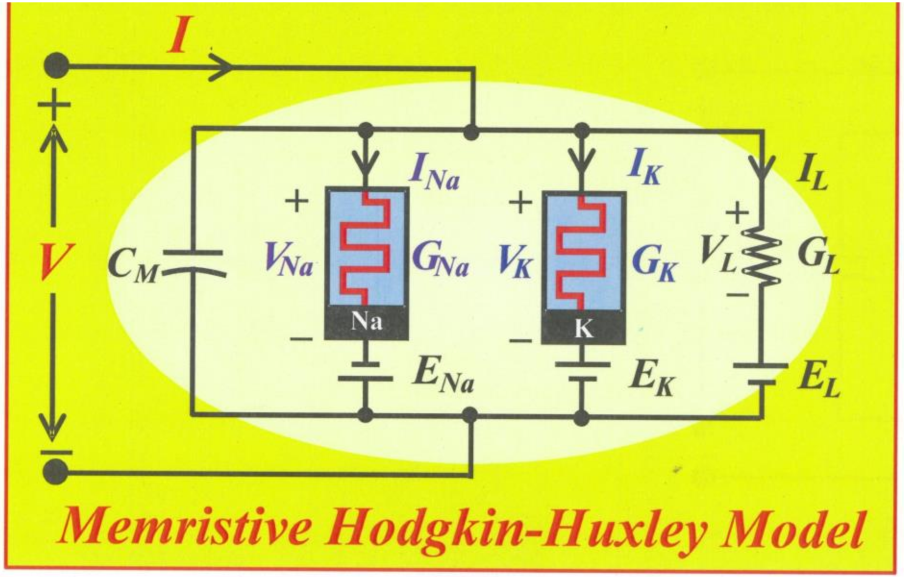
What is a Neuristor?

Postulated by Hewitt Crane in 1960 as an electronic analog of the Hodgkin-Huxley axon.

Previous implementations required inductors

Leon Chua's Revision of the Hodgkin-Huxley Model

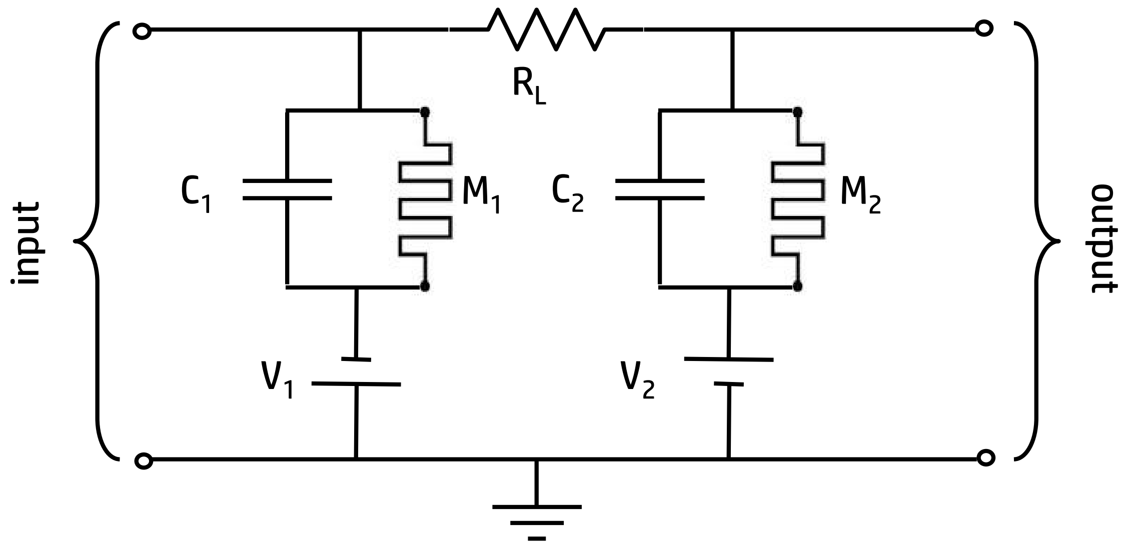
Removes anomalies and paradoxes



L. Chua et al., "Hodgkin-Huxley Axon is made of Memristors,"
International Journal of Bifurcation and Chaos **22** (2012) art. # 1230011.

Neuristor Equivalent Circuit Diagram

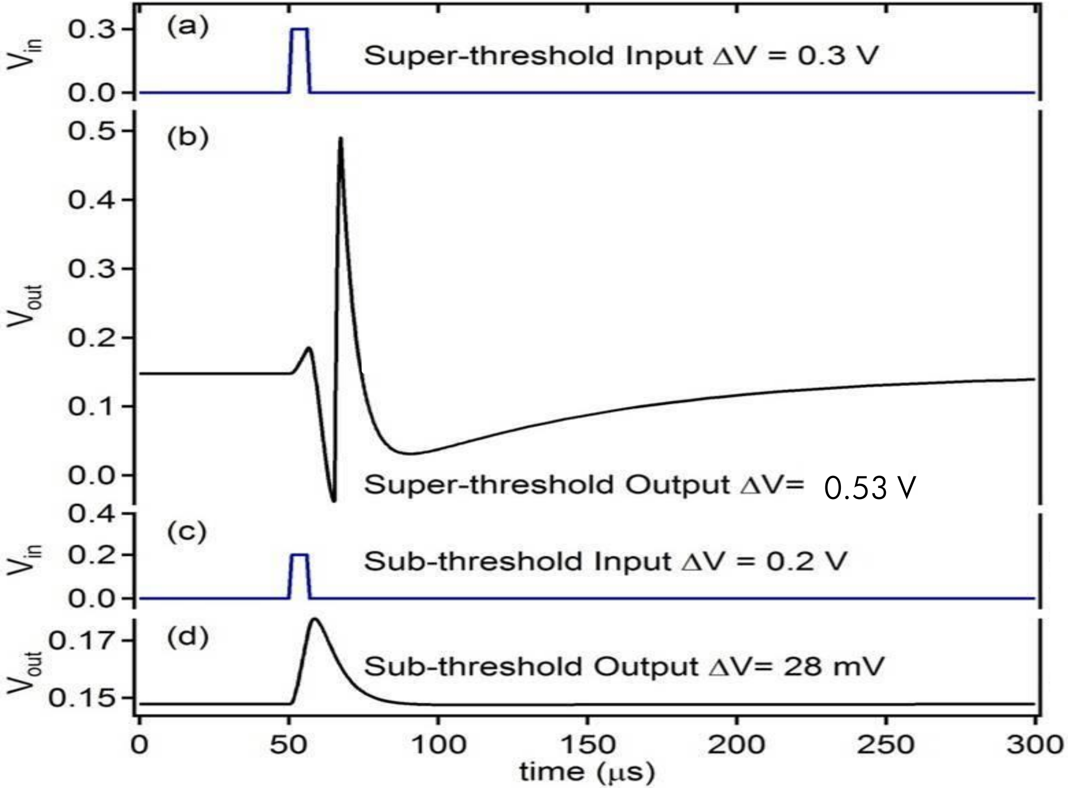
Pickett and Williams, Nature Materials 2013



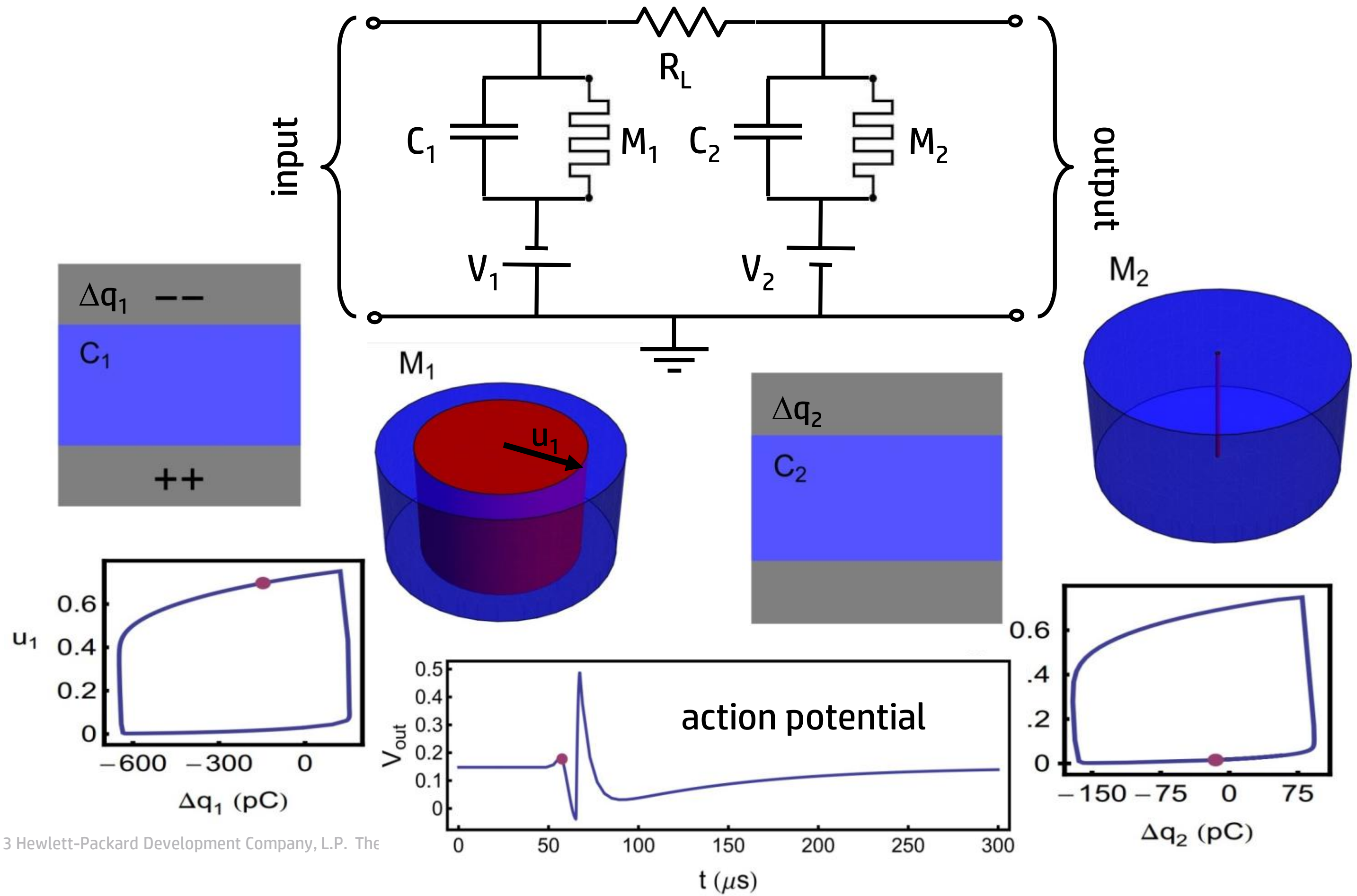
Four state variables: q_1, u_1, q_2, u_2

A. J. Cote proposed the first prototype neuristor in 1961,
and Jin-ichi Nagumo demonstrated an alternate in 1962.
Size of a shoebox. Very little work since.

Neuristor Pulse Amplification and Threshold Behavior

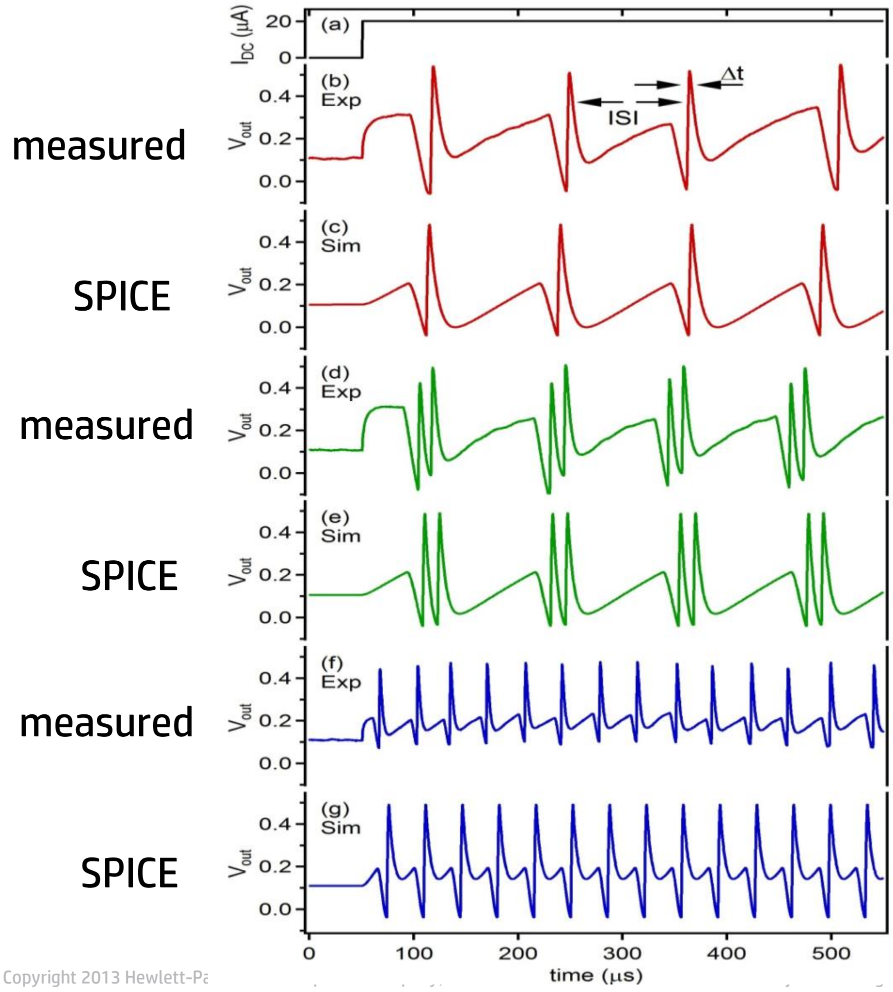


Neuristor Equivalent Circuit Diagram



Neuristor Spiking

emulates signals seen in brains



“Regular Spiking”

C_1=5.1 nF, C_2=0.75 nF

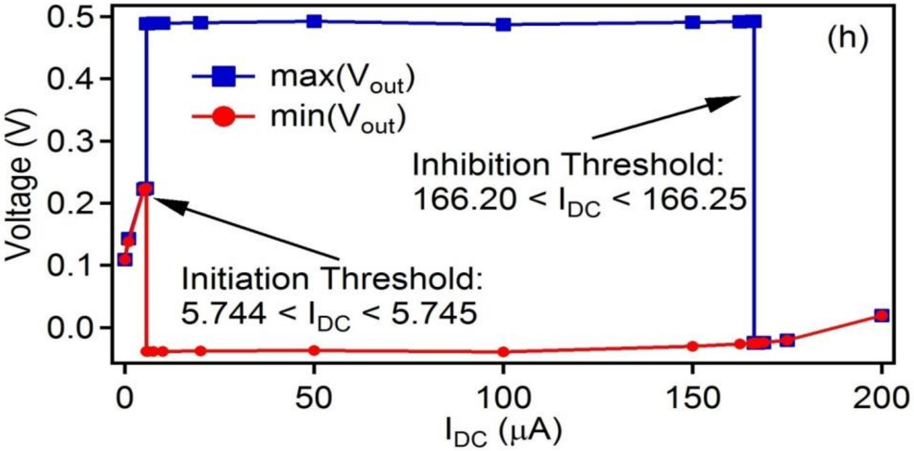
“Chattering”

C_1 = 5.1 nF, C_2 = 0.5 nF

“Fast Spiking”

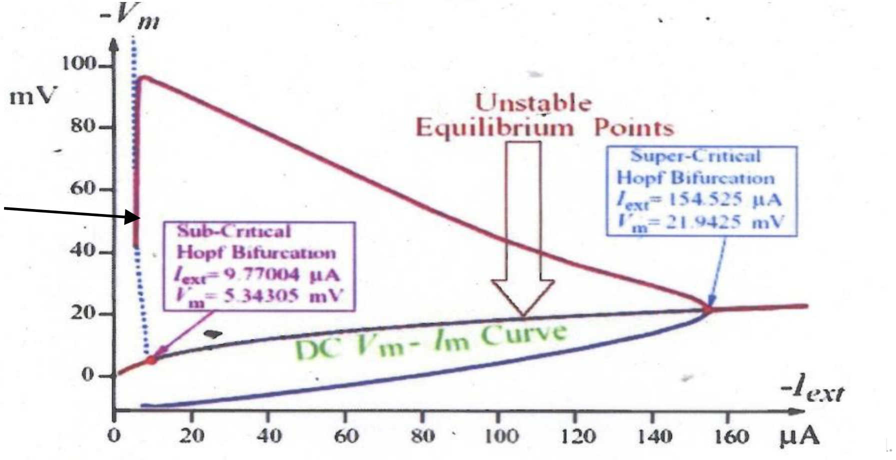
C_1=1.6 nF, C_2=0.5 nF

Compare bifurcation diagrams: neuristor and HH neuron



"fast spiking" neuristor

Chua has noted that biology is poised on the 'edge of chaos' – High information or low power?



HH axon
Chua et al., 2012

Neuristor Summary – an electronic and inorganic HH unit

We have built a *Neuristor* using two *Locally-Active* Memristors and two Capacitors as the Dynamical Elements

The Neuristor has four state variables: the radii of the conductions channels of the memristors and the capacitor charges

The NbO_2 Memristors exhibit an insulator-to-metal transition that mimics the opening and closing of axon ion channels

We have observed signal gain, thresholding, attenuation and spiking that closely mimics the behavior of neuron action potentials

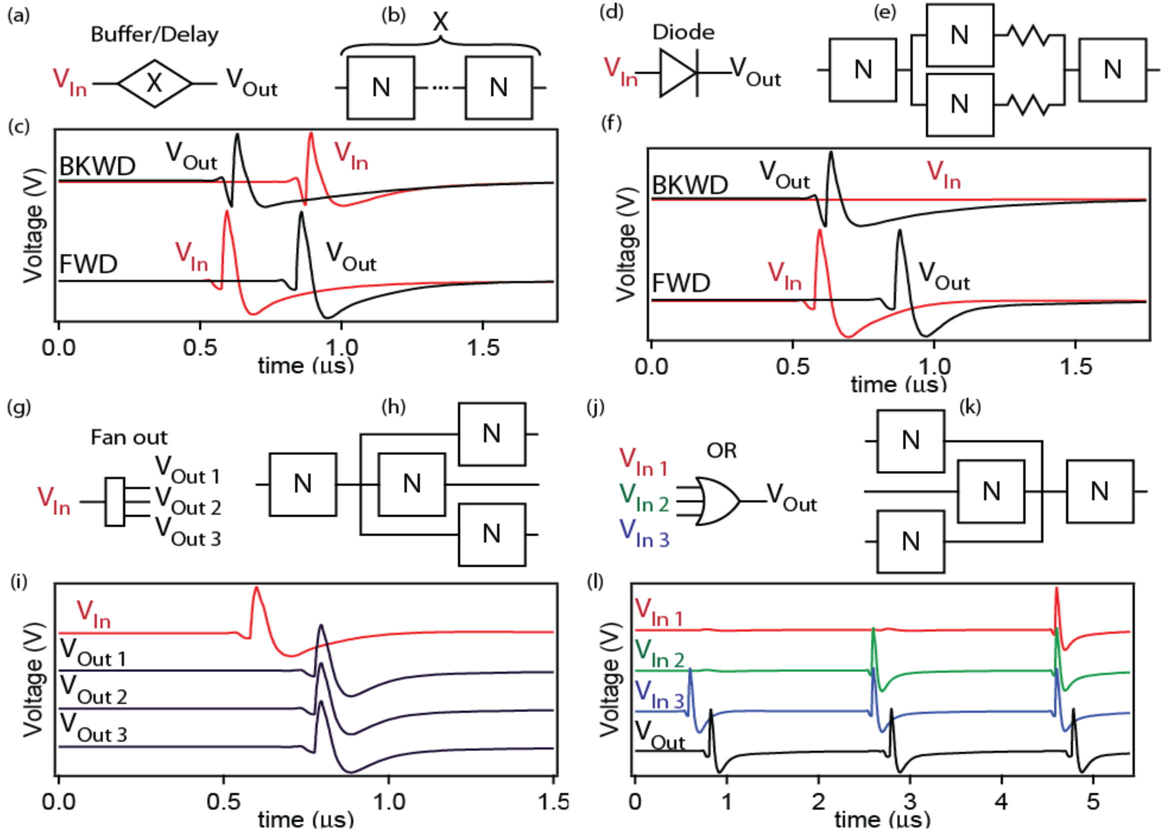
We are designing and simulating transistorless digital logic and analog circuits based on neuristor units using standard CMOS tools.

Neuristor Computation

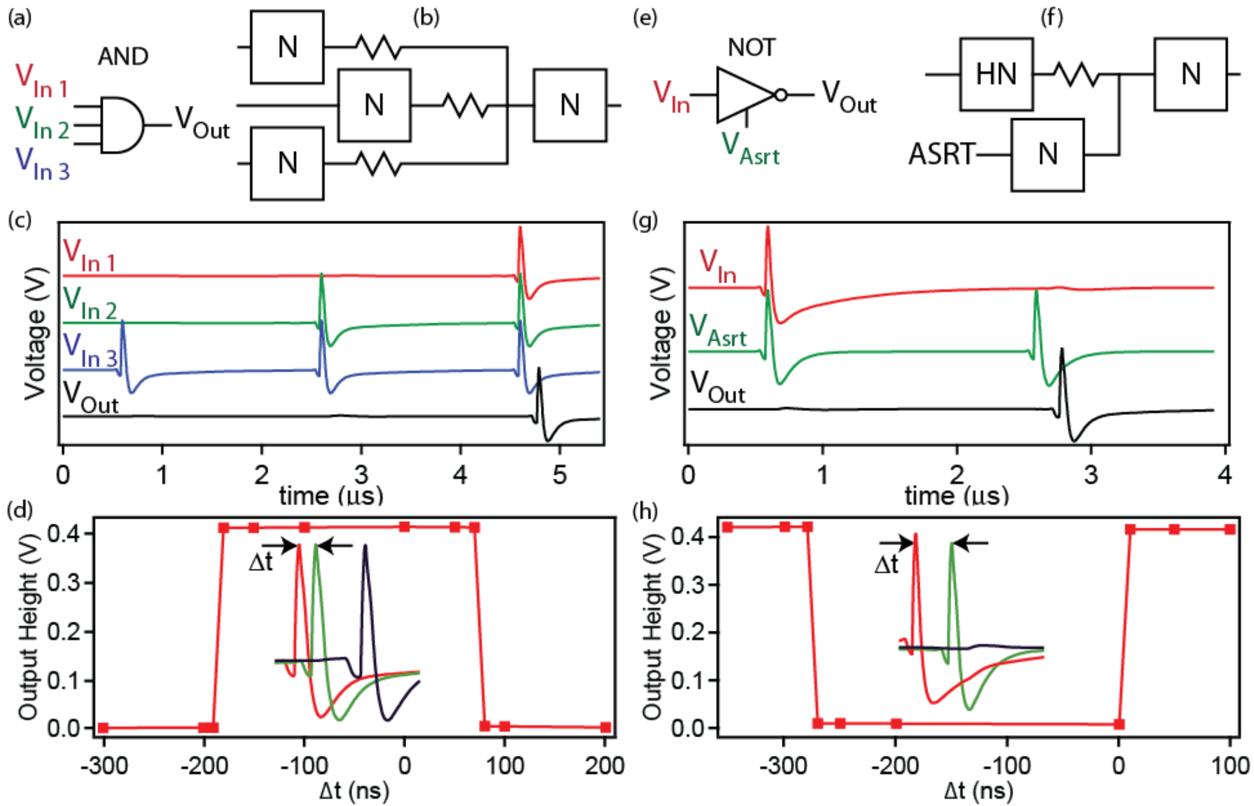
Boolean Logic Gates

Rule 137 Cellular Automaton

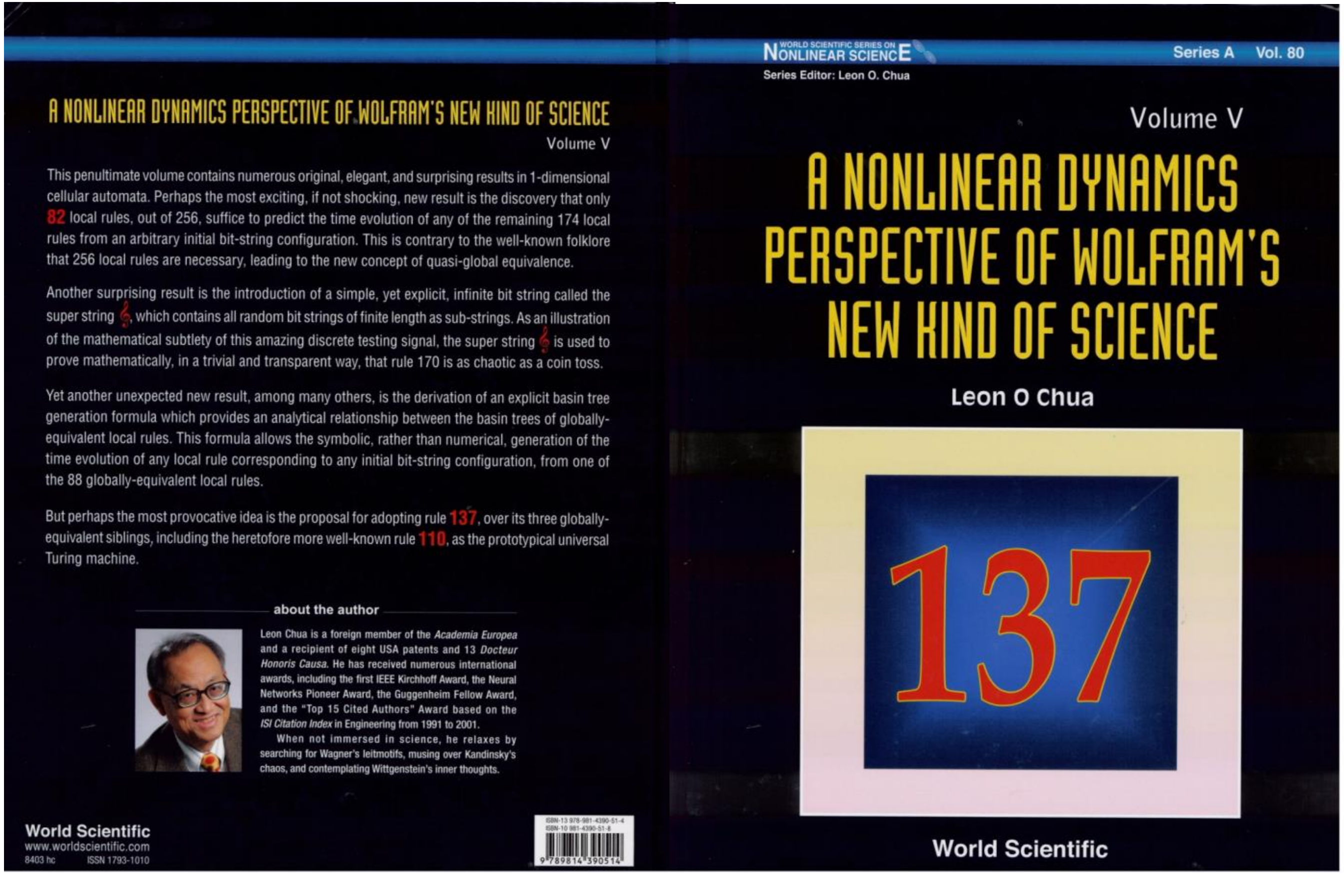
Neuristor Spike-Based Circuit Elements



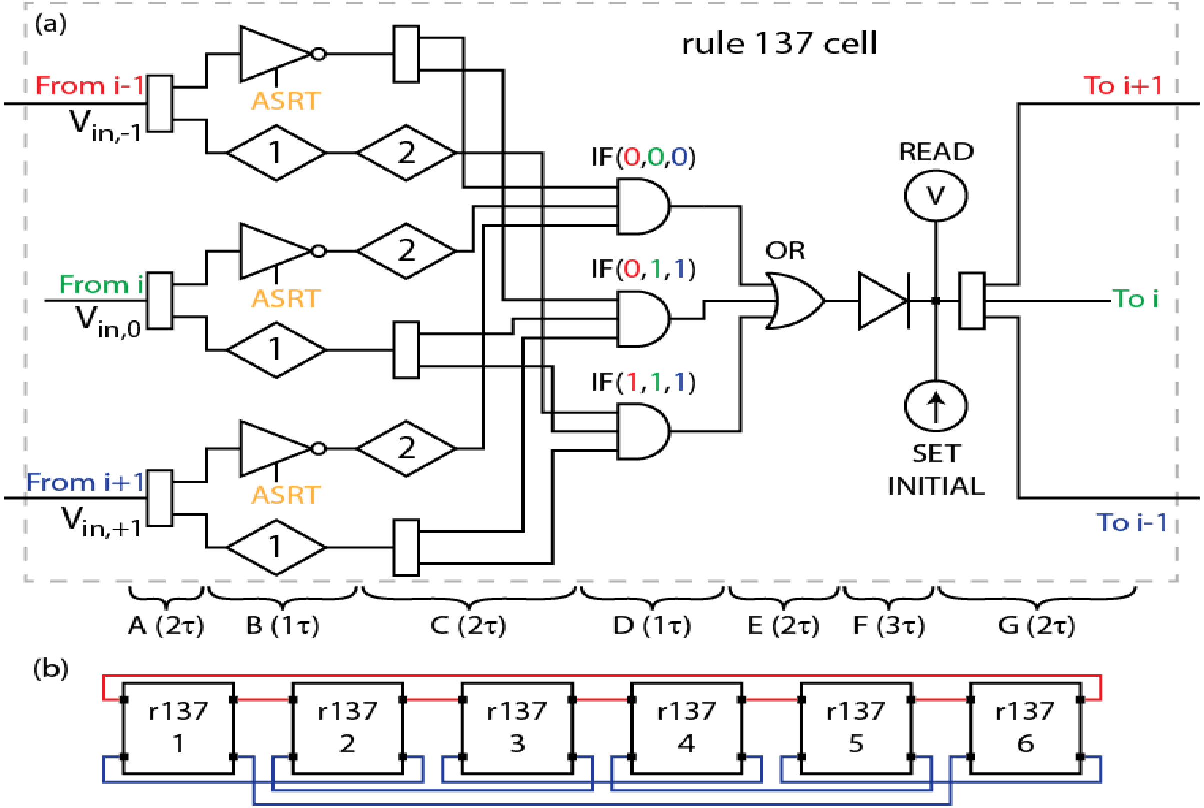
Neuristor Spike-Based AND and NOT Gates



Description of Rule 137 Cellular Automaton



Logical Design of Rule 137 Cellular Automaton

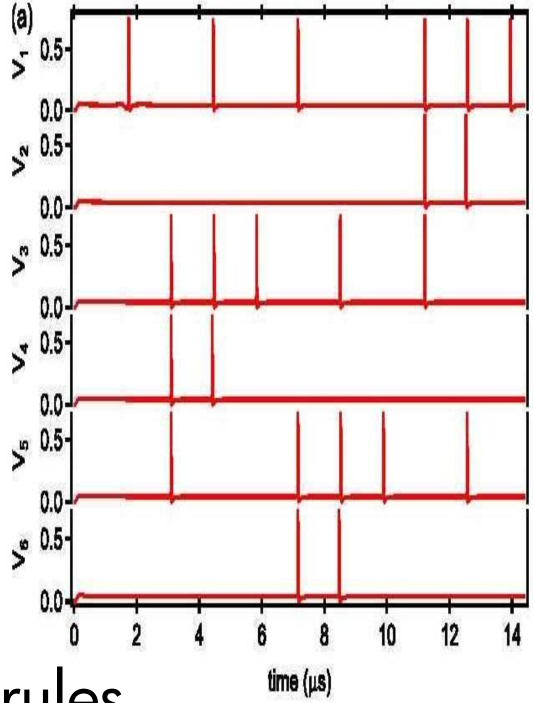


Operation of Rule 137 neuristor cellular automaton

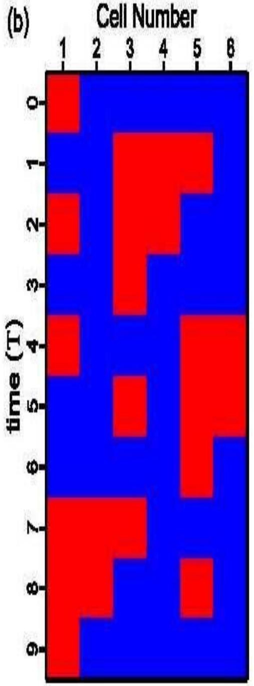
Truth Tables for
CA Rules 110 and 137

$(s_t^{i-1}, s_t^i, s_t^{i+1})$	$F_{110}(\cdot)$	$F_{137}(\cdot)$
(0, 0, 0)	0	1
(0, 0, 1)	1	0
(0, 1, 0)	1	0
(0, 1, 1)	1	1
(1, 0, 0)	0	0
(1, 0, 1)	1	0
(1, 1, 0)	1	0
(1, 1, 1)	0	1

Voltage Spike Timing



Space-Time Diagram



$2^8 = 256$ different rules

Future Research Directions

Understand insulator to conductor transitions!

- Predict materials with desired properties
- Find transition temperature of ~200 C

Neuromimetic integrated circuits –

- Using nanowire capacitance (attoFarads), potential for very high speed and low power circuits - stand-alone or CMOS hybrid

Not attempting to replace Si transitors, but complement them

Q&A